The calculation of inelastic neutrino/dark matter nucleus scattering

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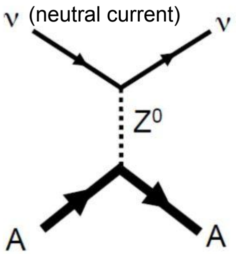
Based on Inelastic nuclear scattering from neutrinos and dark matter (to appear soon)

Coherent elastic neutrino-nucleus scattering

COHERENT @ ORNL: 1 GeV proton beam
CCM @ LANL: 800 MeV proton beam

$$\frac{d\sigma}{dT} = \frac{G_F^2 M}{\pi} F^2(Q) \left[(G_V + G_A)^2 + (G_V - G_A)^2 \left(1 - \frac{T}{E_\nu}\right)^2 - (G_V^2 - G_A^2)\frac{MT}{E_\nu^2} \right]$$



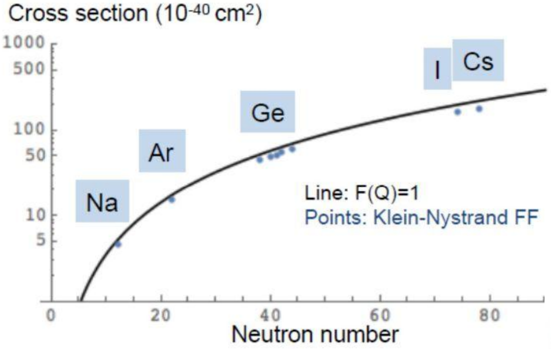
E_ν: Neutrino Energy, T: Nuclear recoil energy, M: Nuclear mass, Q: Momentum transfer

Freedman,'74, Freedman, Schramm, Tubbs,'77

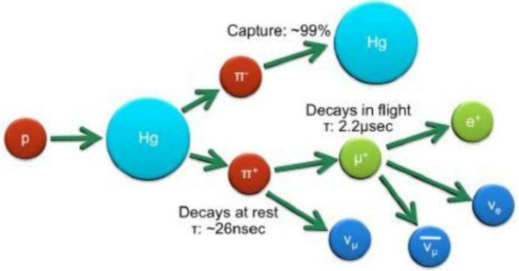
$$\frac{d\sigma}{dT} = \frac{G_F^2 M}{2\pi}\frac{Q_W^2}{4} F^2(Q)\left(2 - \frac{MT}{E_\nu^2}\right)$$

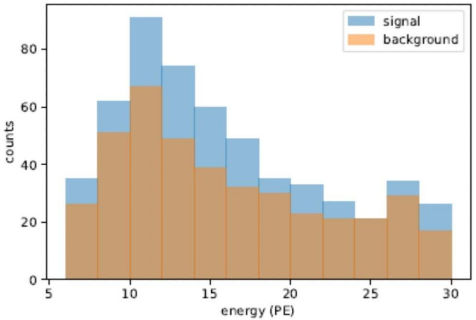
For $T \ll E_\nu$,
$Q_W = N - (1 - 4\sin^2\theta_W) Z$

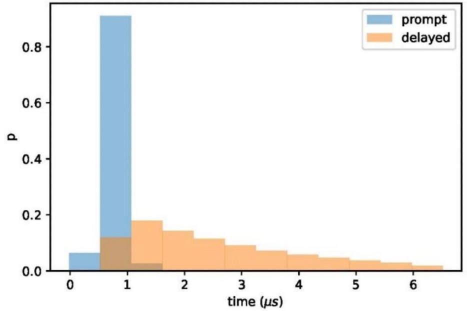
$$\rightarrow \frac{d\sigma}{dT} \propto N^2$$



- **Large Cross-section, but tiny nuclear recoil energy**
- **Detector are now sensitive to ~ keV to 10's of keV recoils**







Prompt: $\pi^+ \rightarrow \mu^+ + \nu_\mu$

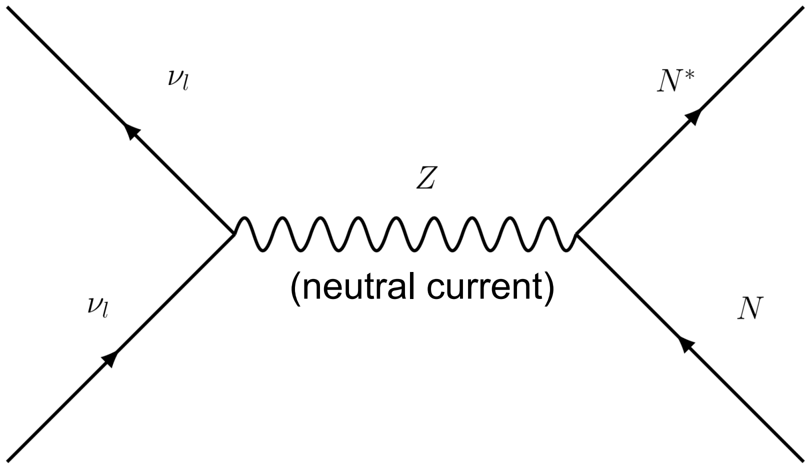
Delayed: $\mu^+ \rightarrow e^+ + \overline{\nu_\mu} + \nu_e$

Inelastic neutrino-nucleus scattering

$$\frac{d\sigma^{\nu}_{inel}}{d\Omega} = \frac{2G_F^2}{\pi(2J+1)} E_f^2 \; W(2m_N E_r)$$

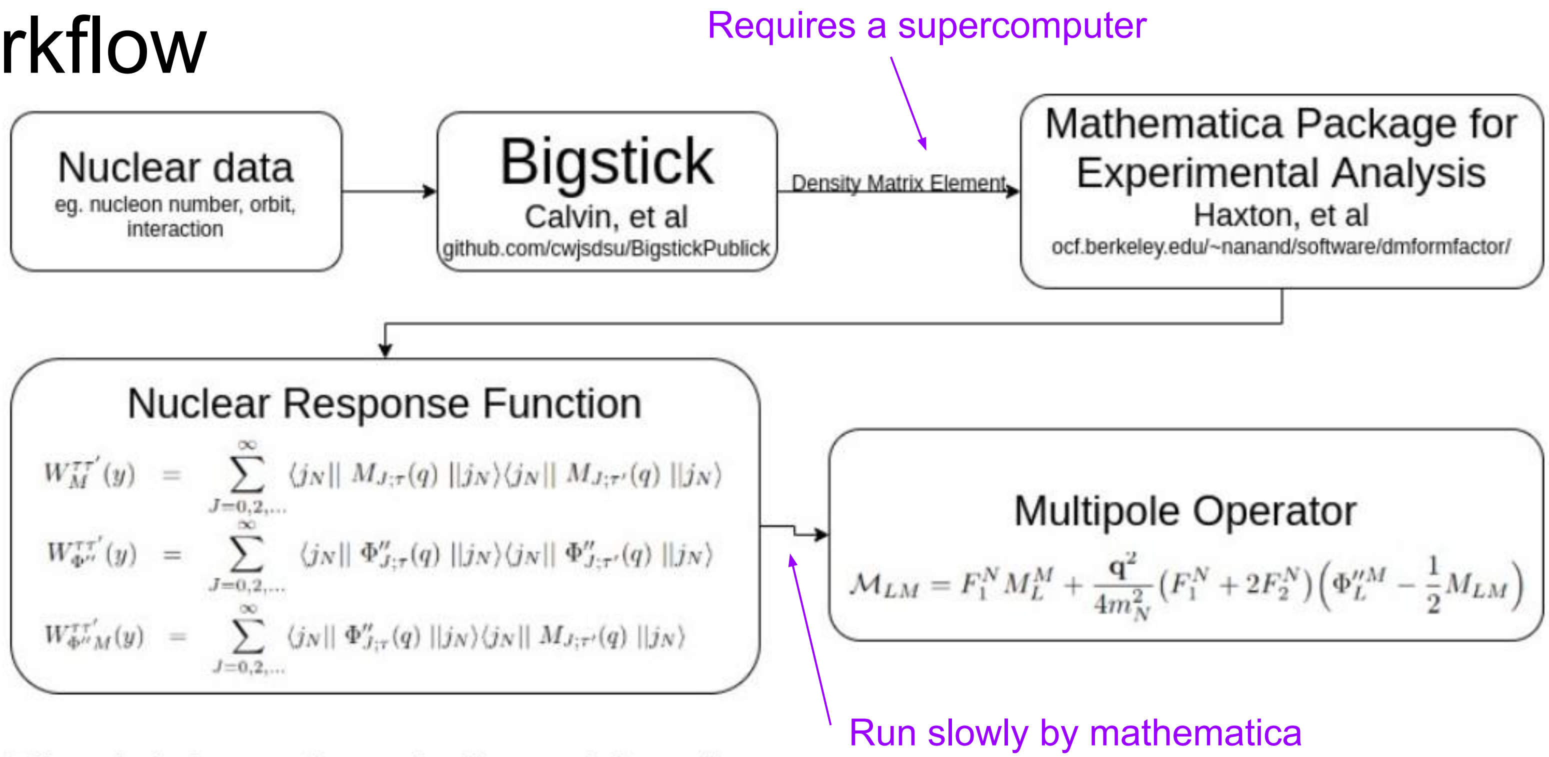
W = linear combination of multipole operators, which is the real challenge

This is crucial to understand the new physics



Walecka, Theoretical Nuclear and Subnuclear Physics, 2004

Workflow



Bigstick is nuclear shell model code.

Anand, Fitzpatrick, Haxton, PRC. 2014

Johnson, Ormand, McElvain, Shan, 2018

Q: Is there a shortcut?

Gamow-Teller operator

Gamow-Teller operator $\frac{1}{2}\hat{\sigma}\hat{\tau}_0$

Strength function

$$\sigma_\nu^{GT} = \frac{G_f^2 g_A^2}{\pi(2J+1)} E_f^2 |\langle J_f || \sum_{i=1}^{A} \frac{1}{2}\hat{\sigma}_i \hat{\tau}_0 || J_i \rangle|^2$$

$$\frac{d\sigma_{inel}^{\nu}}{d\Omega} = \frac{2G_F^2}{\pi(2J+1)} E_f^2 \; W(2m_N E_r)$$

In low recoil energy limit, GT cross section can be related to T^{el5} in multipole analysis.

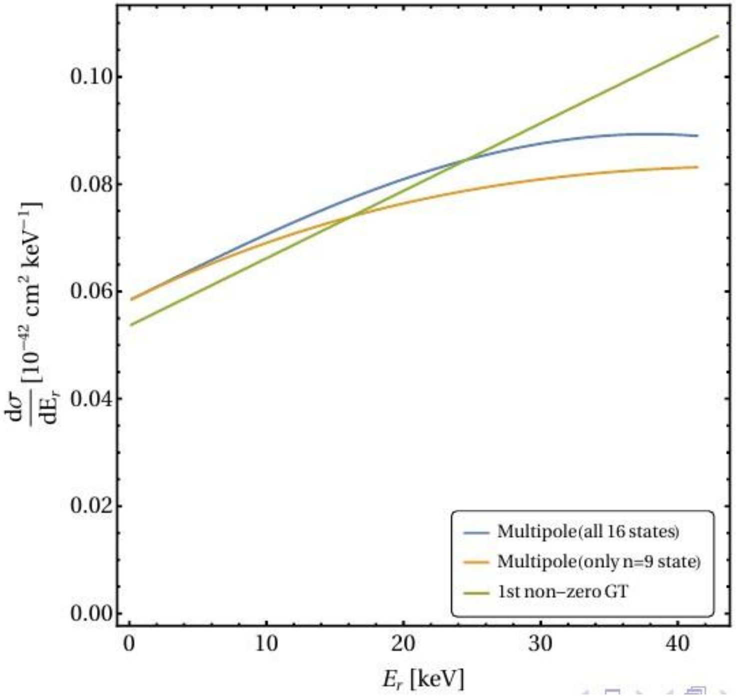
$$\sigma_\nu^{GT} \approx \sigma^{\text{multipole}} (\text{with only } T_{el5}, \text{other terms ignored})$$

Gardiner, PRC, 2021

Multipole vs Gamow-Teller states

We've calculated 16 multipole states (including the ground state), and around 500 states with GT strength (nearly half of them are zero)

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Spin	0	2	4	2	6	2	0	3	1	3	4	5	4	2	0	2



σ^{GT} is just below $\sigma^{multipole}$

GT transitions dominate the inelastic cross section

Q: Is there a shortcut?

A: Yes, GT is the shortcut!

Comparison between two formalisms

Bigstick output

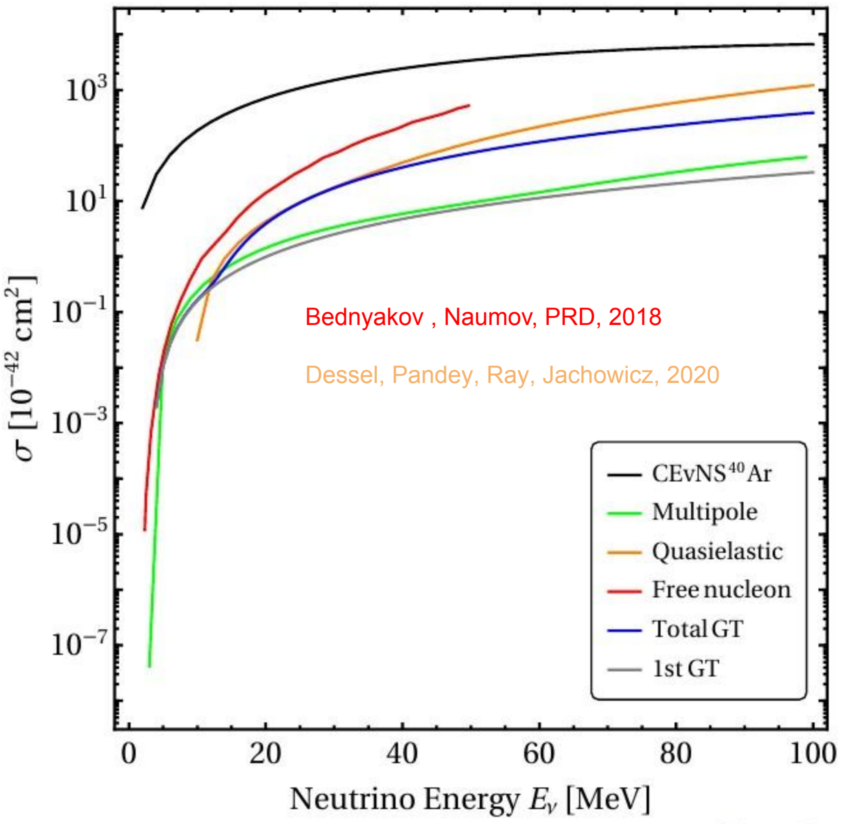
Formalism	Energy	Spin	Isospin	Density matrix	Strength
Multipole	Yes	Yes	Yes	Yes	Yes (via dm)
Gamow-Teller	Yes	No	No	No	Yes

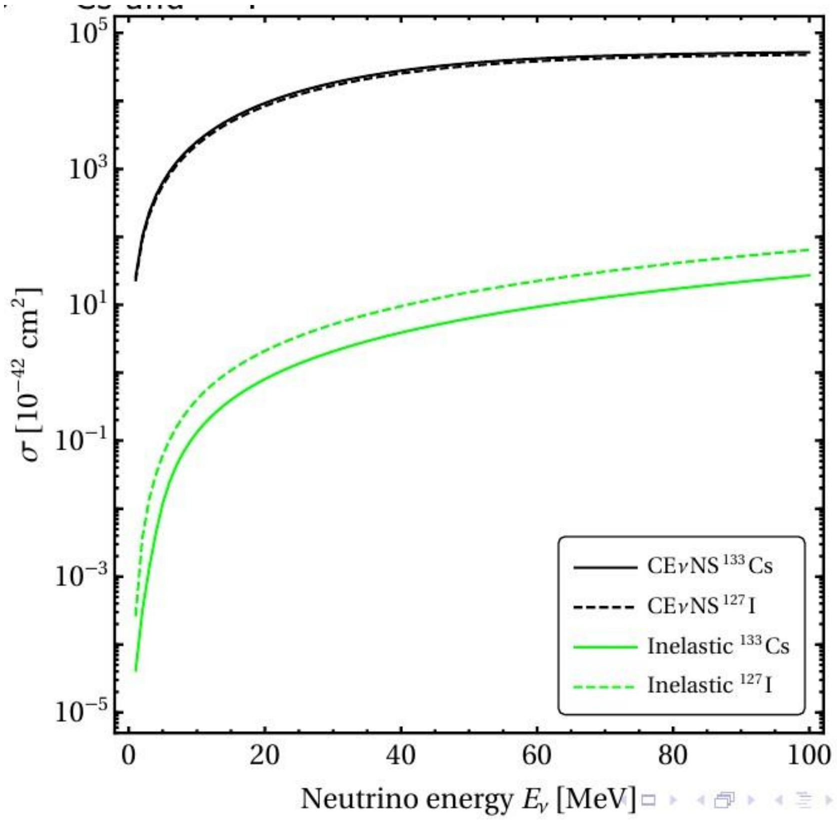
Pros and cons

Formalism	pros	cons
Multipole	Detailed, inclusive	Heavy task
Gamow-Teller	Easy, quick	Less detailed

GT is sufficient for us

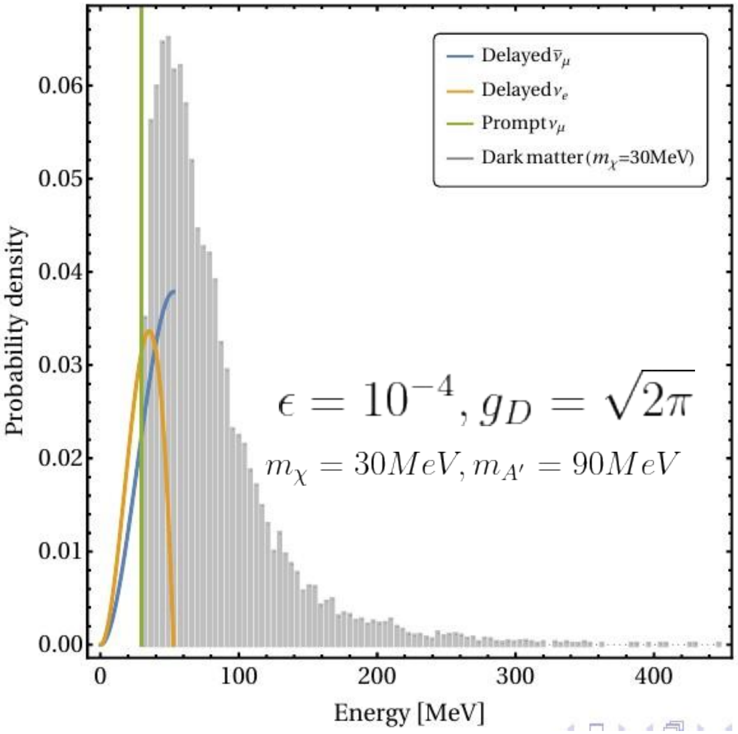
Neutrino nucleus cross section plots





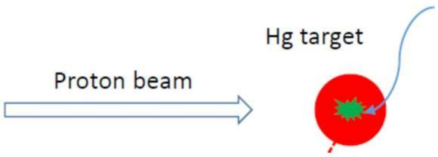
DM-nucleus scattering

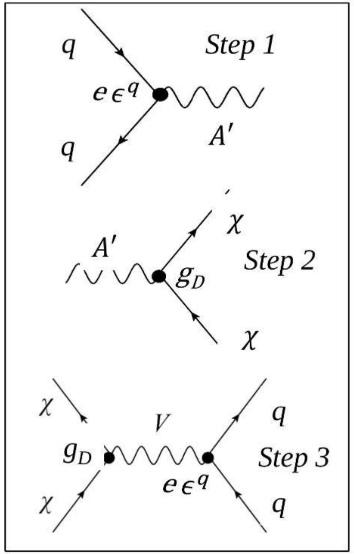
Dark matter energy spectrum



Production of DM(χ_1) at COHERENT

- $\boldsymbol{\pi^0 \to \gamma + A', \ A' \to \chi\chi}$



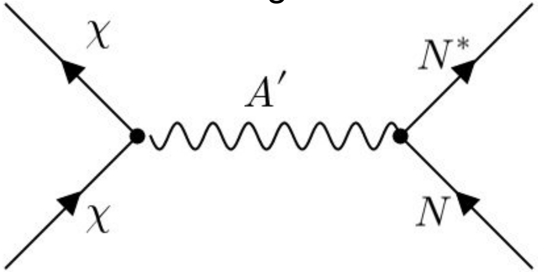


There is also another process: Charge exchange

- $\pi^- + p \to \pi^0 + n$
- $\pi^0 \to \gamma + A'$
- $\pi^{-/+} + p/n \to n/p + A'$
- Dark Bremsstrahlung: $e^{\pm *} \to e^{\pm} + A'$

Lagrangian $\mathcal{L} \supset g_D A'_\mu \bar{\chi}\gamma^\mu \chi + e\epsilon Q_q A'_\mu \bar{q}\gamma^\mu q$

DM-nucleus scattering



deNiverville, Pospelov, Ritz, 2011
Dutta et al., 2020
COHERENT Collaboration, 2019
COHERENT Collaboration, 2021
CCM Collaboration, 2021

Dark matter nucleus scattering

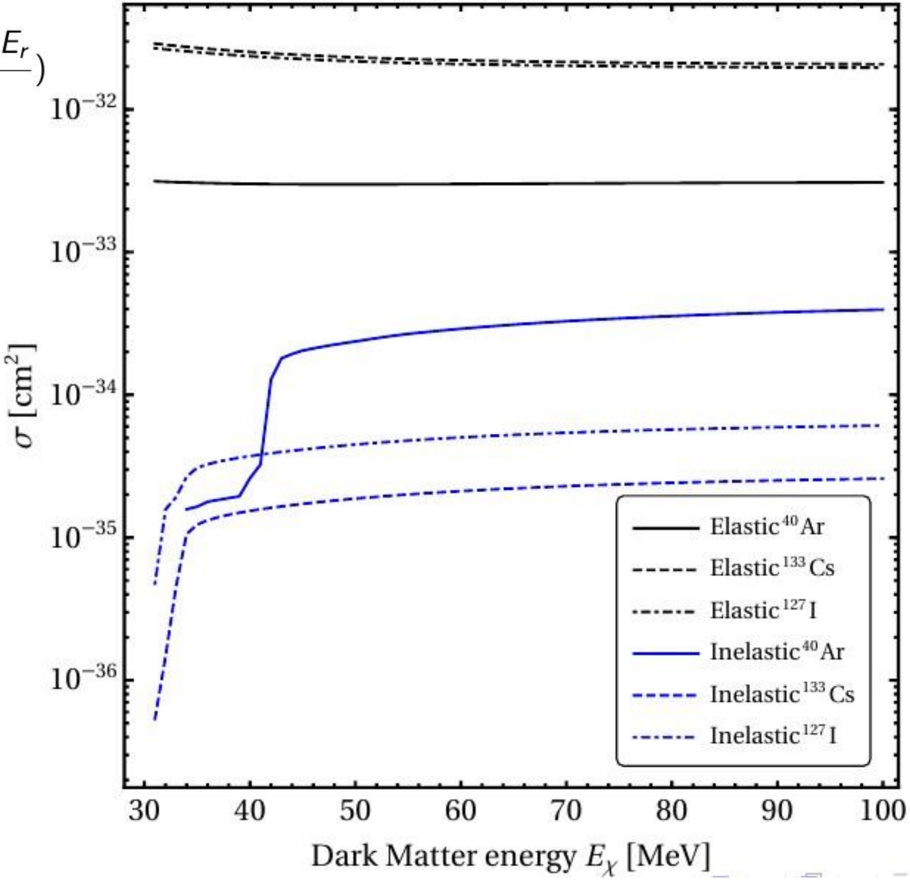
$$\frac{d\sigma}{dE_r}\bigg|_{el} = \frac{e^2\epsilon^2 g_D^2 Z^2}{4\pi(E_\chi^2 - m_\chi^2)(2m_N E_r + m_{A'}^2)^2}\Big[2E_\chi^2 m_N\Big(1 - \frac{E_r}{E_\chi} - \frac{m_N^2 E_r + m_\chi^2 E_r}{2m_N E_\chi^2}\Big) + E_r^2 m_N\Big]|F(2m_N E_r)|^2$$

$$\frac{d\sigma}{dE_r}\bigg|_{inel} = \frac{2e^2\epsilon^2 g_D^2}{p_\chi p'_\chi(2m_N E_r + m_{A'}^2)^2}\frac{m_N}{2\pi}\frac{4\pi}{2J+1}\,|\langle J_f||\sum_{i=1}^{A}\frac{1}{2}\hat{\sigma}_i\hat{\tau}_0||J_i\rangle|^2$$

$$\epsilon = 10^{-4}, g_D = \sqrt{2\pi}$$

$$m_\chi = 30MeV, m_{A'} = 90MeV$$

Total cross section



Event rate ratio

Scattering	Experiment	Elastic / Inelastic
ν-^{40}Ar	COHERENT	364.051
ν-^{40}Ar	CCM	364.051
ν-^{133}Cs	COHERENT	32075.17
ν-^{127}I	COHERENT	2813.77
χ-^{40}Ar (π^0)	CCM	18.76
χ-^{40}Ar (π^-)	CCM	16.61
χ-^{133}Cs	COHERENT	1782.69
χ-^{127}I	COHERENT	685.24

Dark matter has lower ratio as it has higher energy than nu.

Conclusion

- This work is important. One can understand the new physics

- We calculate the inelastic cross section and event rate ratio for neutrino and DM nucleus scattering

- Gamow-Teller transitions dominate the inelastic cross section

- The ratio is smaller in DM scattering because DM has higher energy than nu

- The inelastic contribution is roughly 1% of the elastic

Backup slides

Backup

$$\langle f|\hat{H}_W|i\rangle = \frac{G_F}{\sqrt{2}}\int d^3x \langle f| j_\mu^{lep}\hat{\mathcal{J}}^\mu(\vec{x})|i\rangle$$

$$= \frac{G_F}{\sqrt{2}}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}} \left(l_0 \mathcal{J}^0(\vec{x}) - \vec{l}\cdot\boldsymbol{\mathcal{J}}(\vec{x})\right)$$

Spherical decompsition

Multipole operator $\hat{\mathcal{M}}_{JM}(q) \equiv \int d^3x\,[j_J(qx)Y_{JM}(\Omega_x)]\,\hat{\mathcal{J}}^0(\vec{x})$

Nuclear response function $M_{JM}(q\vec{x}_i) \equiv j_J(qx_i)Y_{JM}(\Omega_{x_i})$

where Y_{JM} is Bessel spherical harmonics

νN scattering: Inelastic cross section (Multipole)

$$W(2m_N E_r) = \sum_{J\geqslant 1, spin} \Big[\frac{1}{2}(\vec{l}\cdot\vec{l}^* - l_3 l_3^*)\left(|\langle J_f||\mathcal{T}^{mag}||J_i\rangle|^2 + |\langle J_f||\mathcal{T}^{el}||J_i\rangle|^2\right)$$

$$- i(\vec{l}\times\vec{l}^*)_3 Re(|\langle J_f||\mathcal{T}^{mag}||J_i\rangle||\langle J_f||\mathcal{T}^{el}||J_i\rangle|)^*\Big] + \sum_{J\geqslant 0, spin}\Big[l_0 l_0^*\,|\langle J_f||\mathcal{M}||J_i\rangle|^2$$

$$+ l_3 l_3^*\,|\langle J_f||\mathcal{L}||J_i\rangle|^2 - 2Re\left(l_3 l_0^*\,|\langle J_f||\mathcal{L}||J_i\rangle||\langle J_f||\mathcal{M}||J_i\rangle|^*\right)\Big]$$

Neutrino current $l_\mu = \bar{\nu}\gamma_\mu\frac{(1-\gamma_5)}{2}\nu$, recoil momentum q

Hoferichter, Menéndez, Schwenk, PRD, 2020

$$\mathcal{M} = \mathcal{M}_{LM} + \mathcal{M}^5_{LM} = \left\{F_1^N M_L^M + \frac{\mathbf{q}^2}{4m_N^2}(F_1^N + 2F_2^N)(\Phi''^M_L - \frac{1}{2}M_{LM})\right\} + \left\{-i\frac{|\mathbf{q}|}{m_N}G_A^N\left[\Omega_L^M + \frac{1}{2}\Sigma''^M_L\right]\right\}$$

$$\mathcal{L} = \mathcal{L}_{LM} + \mathcal{L}^5_{LM} = \left\{\frac{q^0}{|\mathbf{q}|}\mathcal{M}\right\} + \left\{i\left[G_A^N(1 - \frac{\mathbf{q}^2}{8m_N^2}) - \frac{\mathbf{q}^2}{4m_N^2}G_P^N\right]\Sigma''^M_L\right\}$$

$$\mathcal{T}^{el} = \mathcal{T}^{el}_{LM} + \mathcal{L}^{el5}_{LM} = \left\{\frac{|\mathbf{q}|}{m_N}\left[F_1^N\Delta'^M_L + \frac{F_1^N + F_2^N}{2}\Sigma_L^M\right]\right\} + \left\{iG_A^N(1 - \frac{\mathbf{q}^2}{8m_N^2})\Sigma'^M_L\right\}$$

$$\mathcal{T}^{mag} = \mathcal{T}^{mag}_{LM} + \mathcal{L}^{mag5}_{LM} = \left\{-i\frac{|\mathbf{q}|}{m_N}\left[F_1^N\Delta_L^M - \frac{F_1^N + F_2^N}{2}\Sigma'^M_L\right]\right\} + \left\{G_A^N(1 - \frac{\mathbf{q}^2}{8m_N^2})\Sigma_L^M\right\}$$

νN scattering: Inelastic cross section (Multipole)

$$\frac{d\sigma^{\nu}_{inel}}{d\Omega} = \frac{2G_F^2}{\pi(2J+1)} E_f^2 \cos^2\frac{\theta}{2} \Big\{ \sum_{J=0}^{\infty} |\langle J_f || \hat{\mathcal{M}}_J + \frac{q_0}{q}\hat{\mathcal{L}}_J || J_i \rangle|^2$$

$$+ \left[-\frac{q_\mu^2}{2q^2} + \tan^2\frac{\theta}{2} \right] \sum_{J=1}^{\infty} \left[|\langle J_f || \hat{\mathcal{T}}_J^{el} || J_i \rangle|^2 + |\langle J_f || \hat{\mathcal{T}}_J^{mag} || J_i \rangle|^2 \right]$$

$$\mp 2 \tan\frac{\theta}{2} \left[-\frac{q_\mu^2}{q^2} + \tan^2\frac{\theta}{2} \right]^{1/2} \sum_{J=1}^{\infty} Re \left(\langle J_f || \hat{\mathcal{T}}_J^{mag} || J_i \rangle \langle J_f || \hat{\mathcal{T}}_J^{el} || J_i \rangle^* \right) \Big\}$$

Backup Multipole operators

Multipole operators are defined by

$$\begin{aligned}
\hat{\mathcal{M}}_{JM} &= M_{JM} + M^5_{JM} &&= \int d^3x [j_J(qx) Y_{JM}(\Omega_x)] \hat{\mathcal{J}}_0(x) \\
\hat{\mathcal{L}}_{JM} &= L_{JM} + L^5_{JM} &&= \frac{i}{q} \int d^3x \, [\nabla [j_J(qx) Y_{JM}(\Omega_x)]] \cdot \hat{\mathcal{J}}(x) \\
\hat{\mathcal{T}}^{el}_{JM} &= T^{el}_{JM} + T^{el5}_{JM} &&= \frac{1}{q} \int d^3x [\nabla \times j_J(qx) \mathbf{Y}^M_{JJ1}(\Omega_x)] \cdot \hat{\mathcal{J}}(x) \\
\hat{\mathcal{T}}^{mag}_{JM} &= T^{mag}_{JM} + T^{mag5}_{JM} &&= \int d^3x [j_J(qx) \mathbf{Y}^M_{JJ1}(\Omega_x)] \cdot \hat{\mathcal{J}}(x)
\end{aligned}$$

Backup: Nuclear response functions

Nuclear response functions are defined by

$$
\begin{aligned}
M_{JM} &= F_1^{(1)}(q_\mu^2) M_J^M \\
L_{JM} &= \frac{q_0}{q} M_{JM} \\
T_{JM}^{el} &= \frac{q}{m_n} \left(F_1^{(1)}(q_\mu^2) \Delta_J'^M + \tfrac{1}{2}\mu^{(1)}(q_\mu^2) \Sigma_J^M\right) \\
T_{JM}^{mag} &= -\frac{iq}{m_n} \left(F_1^{(1)}(q_\mu^2) \Delta_J^M - \tfrac{1}{2}\mu^{(1)}(q_\mu^2) \Sigma_J'^M\right) \\
M_{JM}^5 &= \frac{iq}{m_n} \left(F_A^{(1)}(q_\mu^2) \Omega_J'^M + \tfrac{1}{2} q_0 F_P^{(1)}(q_\mu^2) \Sigma_J''^M\right) \\
L_{JM}^5 &= i \left(F_A^{(1)}(q_\mu^2) - \frac{q^2}{2m_n} F_P^{(1)}(q_\mu^2) \right) \Sigma_J''^M \\
T_{JM}^{el5} &= i F_A^{(1)}(q_\mu^2) \Sigma_J'^M \\
T_{JM}^{mag5} &= F_A^{(1)}(q_\mu^2) \Sigma_J^M
\end{aligned}
$$

Backup: Nucleon form factor

In previous slide m_n is nucleon mass and $\mu^{(1)}(q_\mu^2) = F_1^{(1)}(q_\mu^2) + 2m_n F_2^{(1)}(q_\mu^2)$. In low recoil energy limit $F_1^{(1)}(0) = 1$, $F_A^{(1)}(0) \sim -1.26$, $F_p^{(1)}(0) = \frac{2M_n F_A^{(1)}(0)}{m_\pi}$, $\mu^{(1)}(0) \sim 4.706$.

We use mathematica package *SevenOperators* (0706.2210) and nuclear shell model code *BIGSTICK*(1303.0905) to calculate the nuclear response functions.

Backup

Dirac form factor $F_1^N = Q^N + \frac{\langle r_1^2 \rangle^N}{6} q^2$

Pauli form factor $F_2^N = \kappa^N$

with charge Q^N, magnetic moment $\kappa^p \approx 1.796, \kappa^n \approx -1.913$

and charge radius $\langle r_1^2 \rangle^N = \langle r_E^2 \rangle^N - \frac{3\kappa_N}{2m_N^2}$

with $\langle r_E^2 \rangle^p \approx 0.707 fm^2, \langle r_E^2 \rangle^n \approx -0.116 fm^2$

Pseudoscalar form factor $G_P = -\frac{4 m_N g_{\pi NN} F_\pi}{q^2 - M_\pi^2} - \frac{2}{3} g_A m_N^2 \langle r_A^2 \rangle$

Axial vector form factor $G_A = \frac{g_A}{(1 - q^2/M_A^2)^2}$ with $F_\pi \approx 92.28 MeV$

$\frac{g_{\pi NN}^2}{4\pi} \approx 13.7, \langle r_A^2 \rangle \approx 0.46 fm^2, g_A \approx 1.276, M_A \approx 1 GeV$

Backup: DM current

Spin sum of DM current $l_\mu = \bar{\chi}\gamma^\mu\chi$ is given by

$$\sum_{s_i,s_f} l_\mu l_\nu^* = \sum_{s_i,s_f} \bar{\chi}(p_f)\gamma^\mu\chi(p_i)\bar{\chi}(p_i)\gamma^\mu\chi(p_f)$$

$$\sum_{s_i,s_f} l_0 l_0^* = 2 - \frac{m_N E_r}{E_\chi^2} + \frac{m_\chi^2}{4E_\chi^2}$$

$$\sum_{s_i,s_f} l_3 l_3^* = 2 - 3\frac{m_N E_r}{E_\chi^2} - \frac{m_\chi^2}{4E_\chi^2}$$

$$\sum_{s_i,s_f} l_3 l_0^* = 2 - \frac{m_N E_r}{E_\chi^2}$$

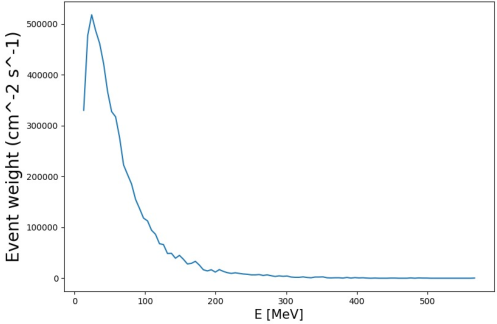
$$\sum_{s_i,s_f} \vec{l}\cdot\vec{l}^* = 2 + \frac{m_N E_r}{E_\chi^2} - \frac{3m_\chi^2}{4E_\chi^2}$$

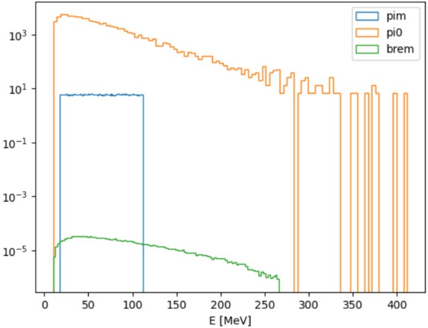
$$\sum_{s_i,s_f} (\vec{l}\times\vec{l}^*)_3 = 0$$

Backup DM Energy spectra

- Coherent Captain-Mills (CCM) experiment (LANL). [800 MeV protons hit a tungsten target, total 7 tons (fiducial) LAr of detector 20m from the target. $\sim 10^{22}$ POT (protons-on-target) per year, currently ongoing] **Aguilar-Arevalo, et al, 2021**
- COHERENT experiment (ORNL) [1 GeV protons hit a mercury target, 14.6kg CsI of detector 19.3m from the target, $\sim 10^{23}$ POT per year] **Akimov, et al, 2017**

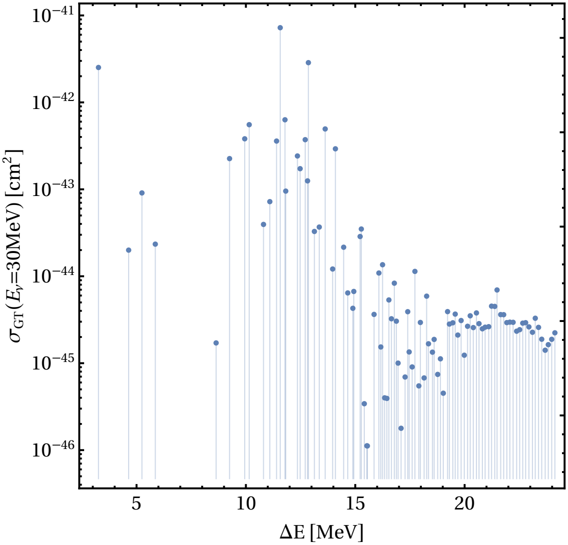
$m_{A'} = 30 MeV$, $m_\chi = 10 MeV$





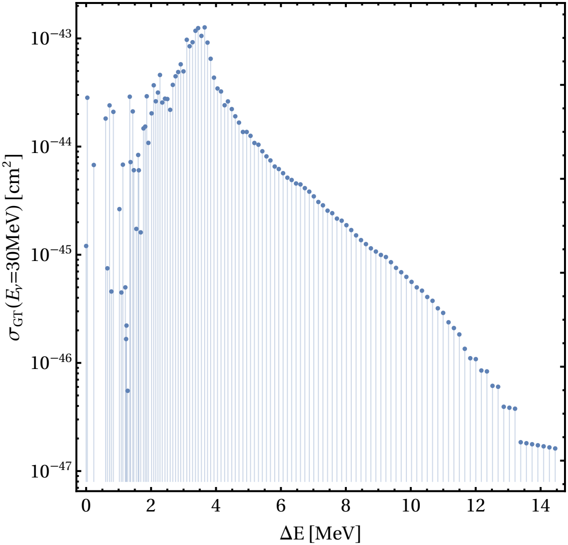
Backup Gamow-Teller transition

^{40}Ar with $E_\nu = 30$MeV



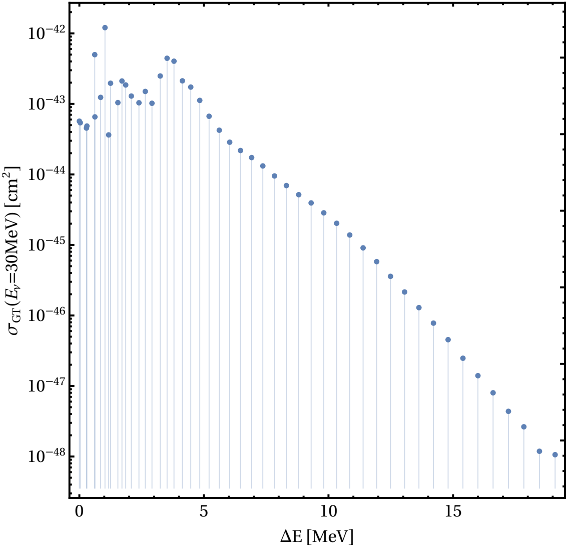
Backup Gamow-Teller transition

^{133}Cs with $E_\nu = 30$MeV

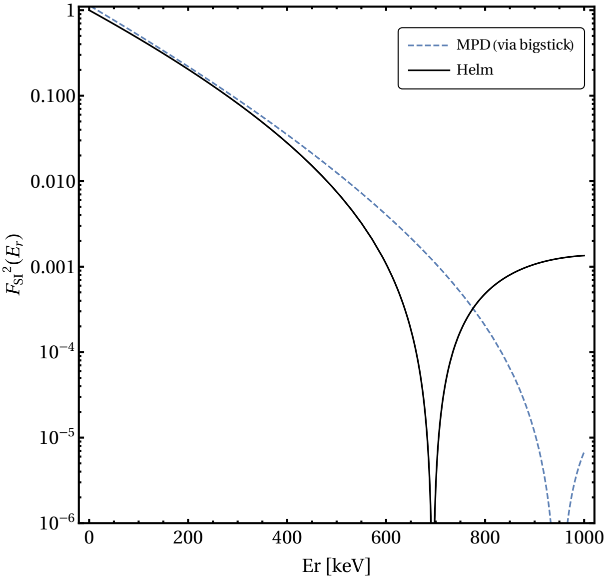


Backup Gamow-Teller transition

^{127}I with $E_\nu = 30$MeV



Backup Helm vs gs to gs: ^{40}Ar



Backup Helm vs gs to gs: ^{133}Cs and ^{127}I